Luzin π -bases and the foliage hybrid operation

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Definition

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 \mathbf{F} = \langle \mathcal{T}, \varphi \rangle \text{ is a foliage tree } : \longleftrightarrow 
  \mathcal{T} \text{ is a tree and } \varphi \text{ is a function with } \text{domain}(\varphi) = \mathcal{T}.
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Recall that the Baire space ${\cal N}$ is ${}^\omega\omega$ with the product topology.

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The standard foliage tree of \mathcal{N} :=

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The standard foliage tree of $\mathcal{N}\ \coloneqq\$ a foliage tree \boldsymbol{S} such that

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$$\mathbf{S} := {}^{<\omega}\omega$$
 and

$$\succ \mathbf{S}_{\mathbf{v}} \coloneqq \{ \mathbf{a} \in {}^{\boldsymbol{\omega}} \boldsymbol{\omega} : \mathbf{a} \text{ begins with } \mathbf{v} \}.$$

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- (3) **F** is **open** in a space $X : \longleftrightarrow$ each **F**_v is an open subset of X.
- f is a **Baire foliage tree** on a space $X : \longleftrightarrow$
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 - (2) F has strict branches,
 - (3) **F** is open in X,
 - (4) skeleton $\mathbf{F} \cong {}^{<\omega}\omega$,
 - (5) $\mathbf{F}_{0_{\mathbf{F}}} = X$ (where $0_{\mathbf{F}} :=$ the least node of skeleton \mathbf{F}).

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Example

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> There is a Baire foliage tree on X.

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Lemma

For any space X the following are equivalent:

- > There is a Baire foliage tree on X.
- > X admits a weaker topology homeomorphic to \mathcal{N} .

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Another Example

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Another Example

There is a Baire foliage tree on the Sorgenfrey line $\mathcal{R}_{\mathcal{S}}$.

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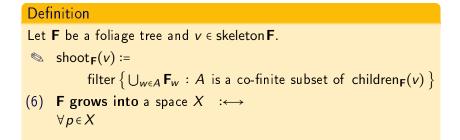
Let **F** be a foliage tree and $v \in \text{skeleton } \mathbf{F}$. $\text{shoot}_{\mathbf{F}}(v) :=$ filter $\{\bigcup_{w \in A} \mathbf{F}_w :$

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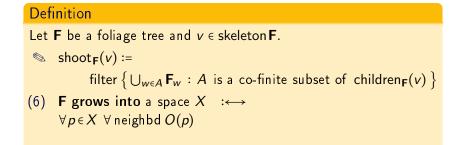
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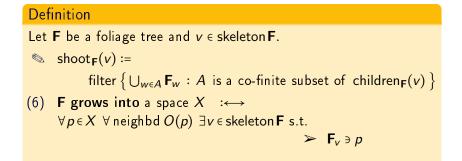
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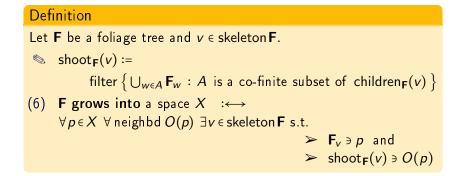


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L is a **Luzin** π -**base** for a space $X : \longleftrightarrow$



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Definition

- **L** is a **Luzin** π -base for a space $X : \longleftrightarrow$
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Examples

 \mathbb{S} is a Luzin π -base for \mathcal{N} .

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Examples

- \mathbb{S} is a Luzin π -base for \mathcal{N} .
- \mathbb{B} is a Luzin π -base for $\mathcal{R}_{\mathcal{S}}$.

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➤ For each $X \in LPB$,

there is a continuous surjection $f: X \xrightarrow{\text{open}} \mathcal{N}$.

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➤ For each $X \in LPB$,

Let LPB := the class of spaces that have a Luzin π -base.

Theorem

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- > For each $X \in LPB$, there is a continuous bijection $g: X \to \mathcal{N}$.
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> For each $X \in LPB$,

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> Up to homeomorphisms, LPB = { $\langle \omega \omega, \tau \rangle : \tau \supseteq \tau_N$ and **S** grows into $\langle \omega \omega, \tau \rangle$ }.

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Examples



The following spaces lie in LPB:

> The irrational Sorgenfrey line $\mathcal{I}_{\mathcal{S}}$;

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 $\succ \quad \text{if } X_{\alpha} \in \{\mathcal{N}, \mathcal{R}_{\mathcal{S}}, \mathcal{I}_{\mathcal{S}}\}$

The following spaces lie in LPB:

- > The irrational Sorgenfrey line $\mathcal{I}_{\mathcal{S}}$;
- ▶ if $X_{\alpha} \in \{\mathcal{N}, \mathcal{R}_{S}, \mathcal{I}_{S}\}$ and $0 < |A| \le \aleph_{0}$,

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The following spaces lie in LPB:

- > The irrational Sorgenfrey line $\mathcal{I}_{\mathcal{S}}$;
- $\begin{array}{ll} \succ & \text{if } X_{\alpha} \in \{\mathcal{N}, \mathcal{R}_{\mathcal{S}}, \mathcal{I}_{\mathcal{S}}\} \text{ and } 0 < |\mathcal{A}| \leq \aleph_{0}, \\ & \text{then } \prod_{\alpha \in \mathcal{A}} X_{\alpha} \in \mathsf{LPB}; \end{array}$

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The following spaces lie in LPB:

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→ if $X \in LPB$, then $X \times \mathcal{N} \in LPB$;

The following spaces lie in LPB:

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- ➢ if X∈LPB, then X × N ∈ LPB;
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- ➢ if X∈LPB, then X × N ∈ LPB;
- if X_α ∈ LPB and 0 < |A| ≤ ℵ₀, then ⊕_{α∈A} X_α ∈ LPB;

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- ➢ if X ∈ LPB, then X × N ∈ LPB;
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- if X_α ∈ LPB and 0 < |A| ≤ ℵ₀, then ⊕_{α∈A} X_α ∈ LPB;
- ▶ if **L** is a Luzin π -base for X and $\emptyset \neq A \subseteq$ skeleton **L**,

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The following spaces lie in LPB:

- > The irrational Sorgenfrey line $\mathcal{I}_{\mathcal{S}}$;
- → if $X \in LPB$, then $X \times \mathcal{N} \in LPB$;
- if X_α ∈ LPB and 0 < |A| ≤ ℵ₀, then ⊕_{α∈A} X_α ∈ LPB;
- if L is a Luzin π-base for X and Ø ≠ A ⊆ skeleton L, then $\bigcup_{z \in A} L_z \in LPB$;

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The following spaces lie in LPB:

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- > if **L** is a Luzin π -base for X and $\emptyset \neq A \subseteq$ skeleton **L**, then $\bigcup_{z \in A} \mathbf{L}_z \in \text{LPB}$;

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➤ if X ∈ LPB

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- if X_α ∈ LPB and 0 < |A| ≤ ℵ₀, then ⊕_{α∈A} X_α ∈ LPB;
- if L is a Luzin π-base for X and Ø ≠ A ⊆ skeleton L, then $\bigcup_{z \in A} L_z \in LPB$;
- ▶ if $X \in LPB$ and $F \subseteq X$ is a σ -compact,

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- if L is a Luzin π-base for X and Ø ≠ A ⊆ skeleton L, then $\bigcup_{z \in A} L_z \in LPB$;
- if X∈LPB and F ⊆ X is a σ-compact, then X \ F ∈ LPB (the proof uses the foliage hybrid operation).

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Definition

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Definition

A tree \mathcal{G} a **graft** for a tree $\mathcal{T} : \longleftrightarrow$



Definition

A tree \mathcal{G} a graft for a tree $\mathcal{T} : \longleftrightarrow$ (1) $\mathcal{G} \cap \mathcal{T} = \{\mathbf{0}_{\mathcal{G}}\} \cup \max \mathcal{G}$

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Definition

A tree \mathcal{G} a graft for a tree $\mathcal{T} : \longleftrightarrow$ (1) $\mathcal{G} \cap \mathcal{T} = \{\mathbf{0}_{\mathcal{G}}\} \cup \max \mathcal{G}$ and (2) $<_{\mathcal{G}} \upharpoonright (\mathcal{G} \cap \mathcal{T}) = <_{\mathcal{T}} \upharpoonright (\mathcal{G} \cap \mathcal{T}).$

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\circledast implant \mathcal{G} := \mathcal{G} \smallsetminus \mathcal{T};
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(2) implant \mathcal{G} := \mathcal{G} \smallsetminus \mathcal{T};

(3) explant (\mathcal{T}, \mathcal{G}) :=
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 $\stackrel{\text{somegative}}{=} \inf \mathcal{G} \smallsetminus \mathcal{T};$
 $\stackrel{\text{somegative}}{=} explant(\mathcal{T}, \mathcal{G}) := (0_{\mathcal{G}}, +\infty)_{\mathcal{T}}$

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 \cong implant $\mathcal{G} := \mathcal{G} \smallsetminus \mathcal{T};$
 \cong explant $(\mathcal{T}, \mathcal{G}) := (0_{\mathcal{G}}, +\infty)_{\mathcal{T}} \smallsetminus [\max \mathcal{G}, +\infty)_{\mathcal{T}}.$

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Definition

A family γ of grafts is **consistent** : \longleftrightarrow

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A family γ of grafts is **consistent** : \longleftrightarrow (3) $\forall \mathcal{E} \neq \mathcal{G} \in \gamma$

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Definition

A family γ of grafts is **consistent** : \longleftrightarrow (3) $\forall \mathcal{E} \neq \mathcal{G} \in \gamma$ [implant $\mathcal{E} \cap \text{implant} \mathcal{G} = \varnothing$]

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A family γ of grafts is **consistent** : \longleftrightarrow (3) $\forall \mathcal{E} \neq \mathcal{G} \in \gamma$ [implant $\mathcal{E} \cap$ implant $\mathcal{G} = \emptyset$] and (4) $\forall \mathcal{E} \neq \mathcal{G} \in \gamma$

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 $\succ 0_{\mathcal{E}} \parallel_{\mathcal{T}} 0_{\mathcal{G}}$

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Definition

A family γ of grafts is **consistent** : \longleftrightarrow (3) $\forall \mathcal{E} \neq \mathcal{G} \in \gamma$ [implant $\mathcal{E} \cap$ implant $\mathcal{G} = \emptyset$] and (4) $\forall \mathcal{E} \neq \mathcal{G} \in \gamma$ $\Rightarrow \quad 0_{\mathcal{E}} \parallel_{\mathcal{T}} 0_{\mathcal{G}} \text{ or}$ $\Rightarrow \quad 0_{\mathcal{E}} \in [\max \mathcal{G}, +\infty)_{\mathcal{T}} \text{ or}$ $\Rightarrow \quad 0_{\mathcal{G}} \in [\max \mathcal{E}, +\infty)_{\mathcal{T}}.$

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Definition

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Hybrid operation

Definition

 $\mathsf{hybrid}(\mathcal{T},\gamma)$ is

Hybrid operation

Definition

 $\mathsf{hybrid}(\mathcal{T},\gamma)$ is a set $\mathcal H$

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hybrid (\mathcal{T}, γ) is a set \mathcal{H} with order $<_{\mathcal{H}}$



Hybrid operation

Definition

hybrid (\mathcal{T}, γ) is a set \mathcal{H} with order $<_{\mathcal{H}}$ such that:



Hybrid operation

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hybrid (\mathcal{T}, γ) is a set \mathcal{H} with order $<_{\mathcal{H}}$ such that:

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hybrid (\mathcal{T}, γ) is a set \mathcal{H} with order $<_{\mathcal{H}}$ such that:

$$\succ \mathcal{H} \coloneqq \left(\mathcal{T} \setminus \bigcup_{\mathcal{G} \in \gamma} \operatorname{explant}(\mathcal{T}, \mathcal{G})\right)$$

hybrid (\mathcal{T}, γ) is a set \mathcal{H} with order $<_{\mathcal{H}}$ such that:

$$\succ \mathcal{H} \coloneqq \left(\mathcal{T} \setminus \bigcup_{\mathcal{G} \in \gamma} \mathsf{explant}(\mathcal{T}, \mathcal{G})\right) \cup \bigcup_{\mathcal{G} \in \gamma} \mathsf{implant}\mathcal{G}$$

hybrid (\mathcal{T}, γ) is a set \mathcal{H} with order $<_{\mathcal{H}}$ such that:

$$\mathcal{H} := \left(\mathcal{T} \setminus \bigcup_{\mathcal{G} \in \gamma} \operatorname{explant}(\mathcal{T}, \mathcal{G}) \right) \cup \bigcup_{\mathcal{G} \in \gamma} \operatorname{implant} \mathcal{G} \text{ and}$$
$$\mathcal{F}_{\mathcal{H}} :=$$

Hybrid operation

Definition

hybrid (\mathcal{T}, γ) is a set \mathcal{H} with order $<_{\mathcal{H}}$ such that:

$$\mathcal{H} := \left(\mathcal{T} \smallsetminus \bigcup_{\mathcal{G} \in \gamma} \mathsf{explant}(\mathcal{T}, \mathcal{G}) \right) \cup \bigcup_{\mathcal{G} \in \gamma} \mathsf{implant}\mathcal{G} \quad \mathsf{and} \\ \mathcal{F} <_{\mathcal{H}} := \mathsf{transitive.closure} \left(<_{\mathcal{T}} \cup \bigcup_{\mathcal{G} \in \gamma} <_{\mathcal{G}} \right).$$

Hybrid operation

Definition

hybrid (\mathcal{T}, γ) is a set \mathcal{H} with order $<_{\mathcal{H}}$ such that:

$$\mathcal{H} := \left(\mathcal{T} \smallsetminus \bigcup_{\mathcal{G} \in \gamma} \mathsf{explant}(\mathcal{T}, \mathcal{G}) \right) \cup \bigcup_{\mathcal{G} \in \gamma} \mathsf{implant}\mathcal{G} \quad \mathsf{and} \\ \mathcal{F} <_{\mathcal{H}} := \mathsf{transitive.closure} \left(<_{\mathcal{T}} \cup \bigcup_{\mathcal{G} \in \gamma} <_{\mathcal{G}} \right).$$

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Proposition

hybrid (\mathcal{T}, γ) is a set \mathcal{H} with order $<_{\mathcal{H}}$ such that:

$$\mathcal{H} := \left(\mathcal{T} \smallsetminus \bigcup_{\mathcal{G} \in \gamma} \mathsf{explant}(\mathcal{T}, \mathcal{G}) \right) \cup \bigcup_{\mathcal{G} \in \gamma} \mathsf{implant}\mathcal{G} \quad \mathsf{and} \\ \mathcal{F} <_{\mathcal{H}} := \mathsf{transitive.closure} \left(<_{\mathcal{T}} \cup \bigcup_{\mathcal{G} \in \gamma} <_{\mathcal{G}} \right).$$

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Proposition

 $\mathsf{hybrid}(\mathcal{T},\gamma)$ is a tree.

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We consider only nonincreasing foliage trees

We consider only nonincreasing foliage trees $(w > v \rightarrow \mathbf{F}_w \subseteq \mathbf{F}_v)$.

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Definition

We consider only nonincreasing foliage trees $(w > v \rightarrow \mathbf{F}_w \subseteq \mathbf{F}_v)$.

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Definition

A foliage tree **G** is a **foliage graft** for a foliage tree **F** : \longleftrightarrow

 \succ skeleton **G** is a graft for skeleton **F**,

We consider only nonincreasing foliage trees $(w > v \rightarrow \mathbf{F}_w \subseteq \mathbf{F}_v)$.

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Definition

- \succ skeleton **G** is a graft for skeleton **F**,
- $\succ \ \boldsymbol{G}_{\boldsymbol{0}_{\boldsymbol{G}}} \subseteq \boldsymbol{F}_{\boldsymbol{0}_{\boldsymbol{G}}},$

We consider only nonincreasing foliage trees $(w > v \rightarrow \mathbf{F}_w \subseteq \mathbf{F}_v)$.

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Definition

- > skeleton **G** is a graft for skeleton \mathbf{F} ,
- $\succ \mathbf{G}_{\mathbf{0}_{\mathbf{G}}} \subseteq \mathbf{F}_{\mathbf{0}_{\mathbf{G}}}, \text{ and }$
- ≻ ∀*m*∈max**G**

We consider only nonincreasing foliage trees $(w > v \rightarrow \mathbf{F}_w \subseteq \mathbf{F}_v)$.

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Definition

- \succ skeleton **G** is a graft for skeleton **F**,
- $\succ \mathbf{G}_{\mathbf{0}_{\mathbf{G}}} \subseteq \mathbf{F}_{\mathbf{0}_{\mathbf{G}}}, \text{ and }$
- $▷ \forall m \in \max \mathbf{G} [\mathbf{G}_m = \mathbf{F}_m].$

We consider only nonincreasing foliage trees $(w > v \rightarrow \mathbf{F}_w \subseteq \mathbf{F}_v)$.

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Definition

- > skeleton **G** is a graft for skeleton \mathbf{F} ,
- \succ $G_{0_G} \subseteq F_{0_G}$, and
- >> \forall *m* ∈ max **G** [**G**_{*m*} = **F**_{*m*}].
- $\operatorname{sut}(\mathbf{F}, \mathbf{G}) \coloneqq$

We consider only nonincreasing foliage trees $(w > v \rightarrow \mathbf{F}_w \subseteq \mathbf{F}_v)$.

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$$\operatorname{cut}(\mathbf{F},\mathbf{G}) \coloneqq \mathbf{F}_{0_{\mathbf{G}}} \setminus \mathbf{G}_{0_{\mathbf{G}}};$$

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- $\operatorname{sut}(\mathbf{F},\mathbf{G}) \coloneqq \mathbf{F}_{\mathbf{0}_{\mathbf{G}}} \setminus \mathbf{G}_{\mathbf{0}_{\mathbf{G}}}$;
- S loss(\mathbf{F}, γ) := $\bigcup \{ \operatorname{cut}(\mathbf{F}, \mathbf{G}) : \mathbf{G} \in \gamma \}.$

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Definition

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Proposition

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- > If **F** and each $\mathbf{G} \in \gamma$ are locally strict, then fol.hybrid(\mathbf{F}, γ) is locally strict.
- If F has strict branches and splittable, and if each G ∈ γ has bounded chains, then fol.hybrid(F, γ) has strict branches.

Let **L** be a Luzin π -base for X

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A foliage graft **G** preserves shoots of **L** : \leftrightarrow $\forall p \in X \ \forall w \in \text{explant}(\mathbf{L}, \mathbf{G}) \text{ s.t. } \mathbf{L}_w \ni p$ $\exists v \in \text{implant} \mathbf{G} \left[\mathbf{G}_v \ni p \text{ and } \text{shoot}_{\mathbf{G}}(v) \gg \text{shoot}_{\mathbf{L}}(w) \right].$

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Proposition

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Proposition

If L grows into X and each $\mathbf{G} \in \gamma$ preserves shoots of L,

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Let **L** be a Luzin π -base for X and let $Y \subseteq X$:

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Definition

A foliage graft **G** preserves shoots of **L** : \longleftrightarrow $\forall p \in X \ \forall w \in explant(\mathbf{L}, \mathbf{G}) \text{ s.t. } \mathbf{L}_w \ni p$ $\exists v \in implant \mathbf{G} \left[\mathbf{G}_v \ni p \text{ and } \operatorname{shoot}_{\mathbf{G}}(v) \gg \operatorname{shoot}_{\mathbf{L}}(w) \right].$

Proposition

If **L** grows into X and each $\mathbf{G} \in \gamma$ preserves shoots of **L**, then fol.hybrid(\mathbf{L}, γ) grows into $X \setminus loss(\mathbf{L}, \gamma)$.

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M. Patrakeev, The complement of a σ -compact subset of a space with a Luzin π -base also has a Luzin π -base, preprint. http://arxiv.org/abs/1512.02458

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Thank you!